On the Unavoidability of the Interpretations of Quantum Mechanics

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Recently, a letter to the Editor "The scandal of Quantum Mechanics" by Prof. van Kampen [13] stimulated an interesting debate [10, 8, 11] on the interpretation of quantum mechanics. The central theme of the debate was a criticism to the presence in the literature (also recent) of voluminous discussions about the interpretation of quantum mechanics. That is what has been called the scandal of quantum mechanics.[13]

We claim that a weak point in the debate was a lack of a definition of the term *interpretation*. In the present note, we would "like to make precise" that meaning and to show how such a clarification is necessary in order to avoid misunderstandings. The concept of interpretation plays a key role in the history of physics so that we hope that the present analysis could be helpful to the student.

Max Jammer [12] in his famous book The philosophy of quantum mechanics distinguishes four different meanings of the word *interpretation*. (See also

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Hempel,[7] Braithwaite,[3] and Suppe.[16]) For what follows, we can restrict our attention to only three different meanings of the word "interpretation", namely (1) partial interpretation of the formalism, (2) interpretation of the theoretical terms, and (3) an interpretation by means of the introduction of a new formalism and then facing (1) and (2) again. We shall now make this explicit.

A physical theory T has at least two components: [7, 3] 1) the formalism F, i.e., the mathematical apparatus of the theory, and 2) the rules of correspondence R which, by means of a meta-language (a language not contained in the formalism), establish the correspondence between the mathematical apparatus and the empirical data under consideration. In particular, when we say that there is a correspondence rule between a term of the formalism and the experimental data, we mean that there is an operational procedure which is used as an interpretation of the term. For example, in classical mechanics the mass of a body is the quantity which is measured by the balance. (See Ref.s [7, 16] for an analysis of the concept of correspondence rules.)

1 Interpretation of the first kind

The introduction of R determines what is called an interpretation of the formalism and for many thinkers this is the only object required to define a theory; i.e., a theory T is a formalism F plus some rules of correspondence R.

It is worth remarking that in classical mechanics as well as in quantum mechanics the formalism F has at least two components: states, s, and observables, A, which are used to define the expectation values $E_s(A)$.

As an example, in the formalism of quantum mechanics based on a Hilbert space H, the observables are self-adjoint operators, A, on H, states are density operators, ρ, on H, and $E_\rho(A) = Tr(\rho A)$. The pure states are in a one-to-one correspondence with the vectors, $\psi \in H$, $\|\psi\| = 1$, and then $E_\rho(A) = \langle \psi, A\psi \rangle$. This is now interpreted (the Born statistical interpretation) as the expectation value in the statistical sense of the observable A in the pure state determined by ψ. The correspondence rule associated to $\langle \psi, A\psi \rangle$ is the following: $\langle \psi, A\psi \rangle$ is the value one obtains when one realizes several measurements of the system by means of an apparatus $\mathcal{A}$ (which correspond to A) and calculate the statistical value of the measurement outcomes.

But it is worth noticing that the Born statistical interpretation by itself does not provide an interpretation of ψ and A; i.e., it does not establish a direct correspondence either between ψ and some empirical data or between A and some empirical data. It only assures that the term $\langle \psi, A\psi \rangle$ is interpreted. We

will return to this problem later while for the moment we limit ourselves to remarking that all the terms of the theory for which we do not have a rule of correspondence (or in other words, which are not directly interpreted) are called *theoretical terms*. Examples are *electron* and *wave function* ψ in the formalism F above with the Born interpretation.

Likewise one may interpret the physical state s of a physical system as the set of the values $E_s(A)$ for all observables, A. Or one may interpret an observable A as the connection between states s and outputs $E_s(A)$. Thus, in this trinity (states, observables, expectation values) one may interpret any one in terms of the other two. [17, 14] What one may not do, at risk of being called illogical, is to interpret all three of the trinity at the same time.

2 Interpretation of the second kind

The above analysis of the structure of an empirical theory points out that the interpretation of the formalism is mandatory for a satisfactory formulation of quantum mechanics. That means that we need to choose some elements of the formalism and to explain what those elements mean physically (by means of rules of correspondence). Once an interpretation of the formalism is introduced we have a (partially) interpreted theory which may contain some theoretical terms. But what about the theoretical terms? One could try to assign to them some physical meaning as we do when we say that the wave function describes the state of an individual particle or the state of an ensemble of particles or when we say that it describes a matter field or a pilot wave, etc. All such approaches (although when suggested by the formalism) have some degree of arbitrariness and correspond to different interpretations of quantum mechanics in a second sense which is linked to the interpretation of the theoretical terms.

3 Interpretation of the third kind

A third kind of "interpretation" occurs when the formalism F is replaced by a different formalism F' to which there corresponds a new set of rules of correspondence R'. Some examples are the formulations of quantum mechanics suggested by Bohm,[4] Ghirardi-Rimini-Weber,[6] and the phase space formulation of quantum mechanics [17, 1, 2] (to be distinguished from the Wigner formulation). The phase space formulation provides a differential geometric foundations of quantum mechanics and allows a derivation of classical and quantum mechanics in a unique setting. [1, 2] In a forthcoming paper, we will

review the phase space formulation and try to catch its possible interpretations.

It is worth remarking that this third kind of interpretation is a reformulation of the formalism with new rules of correspondence. That is important since the fact that it brings about a new "interpretation" is a consequence of the fact that it brings about a new *formalism.* It should be pointed out that the need for introducing the new formalism is the search for a full interpretation, that is, an interpretation of the second kind of the new formalism (given that an analogous interpretation was problematic for the old formalism). Usually, such a strategy is inspired by a realistic position towards the physical theories (i.e., a physical theory describes some elements of the physical reality) and is motivated by the fact that, in this realist perspective, the new formalism describes the same veiled reality described by the old one. An example is the Bohm formulation of quantum mechanics where it is possible to interpret the wave function as a pilot wave.

4 On the unavoidability of the interpretation of quantum mechanics

As we have seen an empirical theory T is divided into a formalism F and a system of rules of correspondence R. Since F without R would be a mathematical theory void of any empirical content, the first kind of interpretation is unavoidable.

What about the second kind of interpretation, i.e., the interpretation of the theoretical terms? One way to avoid the problem of the theoretical terms is to limit ourselves to a minimal interpretation of the formalism such as the minimal statistical interpretation (where we interpret only the term $\langle \phi, A\psi \rangle$) avoiding the problem of the interpretation of the theoretical terms. This corresponds to an instrumentalist approach where quantum mechanics (or more generally a physical theory) is only an instrument to make previsions about the physical phenomena and is not a description of a veiled reality. Clearly, in the instrumentalist framework all the interpretative problems of quantum mechanics ("What is the physical meaning of the wave functions," "What should the physical interpretation of the indeterminacy inequality for position and momentum be?", etc.) that are at the origin of the studies about the interpretation of quantum mechanics disappear at once. Thereby, in the instrumentalist view one does not need to resort to the macroscopic nature of the measurement instruments in order to explain the measurement problem

since there is no measurement problem!
That seems to be the choice made by Henry [8] but not the choice made by Hobson. [10] It is not clear to us if it is the path followed by van Kampen since the sentence "the entanglement between two electrons is a manifestation of the wave function" in van Kampen's letter seems to suggest an interpretation of the wave function and then an interpretation of a theoretical term and then an interpretation of the theory! Indeed, that is what happens in the letter by Hobson [10] where the expression "manifestation of the wave function" is replaced by the expression "manifestation of a matter field." Therefore, Hobson [10] is not rejecting the 'trend" of interpreting quantum mechanics but is proposing his personal interpretation (of the second kind) of quantum mechanics (the wave function represents a matter field) as more powerful because (in his view) it is not an interpretation!

It remains to analyze the third kind of interpretation. As we pointed out, it implies the commitment with a non standard formalism and this seems to be the main target of the critics contained in the "letters to the Editor", i.e., the scandal of quantum mechanics. [13, 10, 11] Anyway, it is worth remarking that an interpretation of the third kind is avoidable only if one supposes that the standard formalism of quantum mechanics is the better one or is the only we need, but such a viewpoint contains some kind of metaphysical assumption.

The motivations for having a non-standard formalism are some reasons related to the standard formalism not being suitable to answer some particular questions as for example: the attribution of properties to an underlying reality, the problem of establishing if the wave function describes a single system or an ensemble of systems, the problem of looking for a formalism that contains a kind of inequality which can be interpreted as limiting the accuracy of a simultaneous measurement of position and momentum,[1] the problem of the relationships between classical and quantum mechanics.
We used the Bohm interpretation as an example of interpretation of the third kind. Now, we would like to remark that Bohm's 1952 papers "should be not taken as his dramatic conversion to a deterministic, mechanical viewpoint. He was merely trying to show that an alternative (to the Copenhagen interpretation) that attributed properties to an underlying reality was possible." (See

[1]We recall that the Heisenberg inequality cannot be interpreted in that sense but only in the sense of a limit to the accuracy on independent measurements of position and momentum. Such an interpretation is instead possible in the phase space approach [17] where observables are not described by self-adjoint operators but by Positive Operator Valued Measures (unsharp observables) and a joint observable for the unsharp position and momentum exists.

Ref. [9], page 7). "The theory was not proposing the existence of a classical 'rock-like' particle, but rather a new kind of entity which is quite different from a classical particle." (Ref. [9], page 9.) Therefore, Bohm's approach was inspired by a realist's position toward quantum mechanics; i.e., quantum mechanics describes some element of reality. In other words, the Bohm formulation of quantum mechanics was developed in order to get an interpretation of the second kind: i.e., an interpretation of the theoretical terms. Then, we could say that, in a certain sense, Bohm and Hobson are aiming at the same target.

In our opinion a clarification of the concept of *interpretation* should be preliminary to any critics to the interpretations of quantum mechanics. Following our distinction between the different concepts of *interpretation*, that is, partial interpretation of the formalism, interpretation of the theoretical terms and interpretation by means of the introduction of a new formalism we can say that all the authors of the letters to the Editor cited above[13, 10, 11, 8] are giving an interpretation of quantum mechanics. In particular, Hobson is giving an interpretation of the second kind of the formalism of quantum field theory while Henry [8] asserts that quantum mechanics does not refer to an external physical reality and that "the wave function is merely an intellectual tool used by physicists." There is some characteristic of an instrumentalist interpretation (which is an interpretation of the first kind) in his view.

It is very interesting to notice that in Ref.s [10, 11] it is not taken into account the fact that once we have introduced an interpretation of a theoretical term (not in the sense of introducing a rule of correspondence) we have also proposed an interpretation of the theory since the interpretation of the theoretical terms (for example the interpretation of the wave function as a matter field) does not come directly from the experimental data (although they are in agreement). That becomes even more interesting if one realizes that usually an interpretation of the second kind is connected with a realist's approach to the physical theories. Perhaps that is due to the fact that physicists are so well aware of the heuristic power of the interpretation of the theoretical terms that, in a certain sense, they see it as part of the formalism. Anyway, that is an error from the epistemological viewpoint. All that makes even less understandable why an interpretation of the third kind should be scandalous! Indeed, the latter is a tentative grasping of a piece of reality and it is not so clear why it must be (a priori) less successful than standard quantum mechanics in this endeavor.

5 Empiricist interpretation

In the present section we would like to give a sketch of an interpretation of the first kind of quantum mechanics which is open to possible interpretations of the second kind. It rests on the work of Ludwig [14] and was developed by de Muynk [15] who called it "empiricist interpretation of quantum mechanics." We already noticed that the main element of the formalism of a physical theory are *states* and *observables*. It is possible to introduce observational terms corresponding to the quantum state ψ and the quantum observable A. That can be done by introducing the concept of preparation apparatuses (corresponding to ψ) as distinguished from the measurement apparatuses (corresponding to A). (See Ref. [14] for a rigorous treatment.) A preparation apparatus is used in order to prepare the system which subsequently will interact with the measurement apparatus. In its turn the measurement apparatus is characterized by a pointer whose position after the measurement determines the measurement outcome.[2] That leads to an empiricist's interpretation [15] where quantum mechanics is seen as a theory describing correlations between macroscopic objects (preparation and measurement apparatuses). Clearly in this framework all the paradoxical aspects of quantum mechanics are absent since there is no direct reference to any microscopic object although its existence is not denied. (See Ref. [15], page 75.)

6 Conclusions

In our opinion the empiricist's interpretation is the starting point for any reasoning about the physical content of quantum mechanics. There are several options that a physicist may follow: 1) to stick with the empiricist's interpretation which in some sense is minimal, 2) to opt for a realist's approach which implies the necessity of an interpretation of one (or two) of the concepts state, observable, and measurement, referring to an existing physical entity, 3) to try to change the formalism.

[2] It is worth remarking that, in a real experiment, the pointer does not point toward a point but toward an interval. Indeed, the intrinsic fuzziness of the measurement process which is in general unavoidable must be taken into consideration. That is perfectly contemplated if one uses Positive Operator Valued Measures in order to describe the measurement process [17, 5].

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